CASCADE CHARACTER OF THE GROWTH OF TURBULENT EDDIES

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The development of discrete large-scale eddies in a turbulent shear flow is examined.

The development of turbulent eddies is a complex phenomenon which combines features of discrete and continuous processes and is "on the boundary" between deterministic and random processes. As was noted in [1], in analyzing complicated objects on which empirical data is limited, the most fruitful approach is constructing simplified theoretical models which successively approximate the phenomenon. However, this approach has not been widely used in turbulence theory.

Reynolds [2], having proposed that the dynamic characteristics of a flow be regarded as the sum of the mean and fluctuation components, actually introduced a turbulence model in the form of disordered, random perturbations superimposed on the main flow. Somewhat later the attention of investigators focused on the presence of characteristic discrete structures, or turbulent eddies, in a turbulent flow. The scale of these structures changes across the flow. The hypothesis evidently first advanced by Richardson in 1922 [3] was used for several years to describe this phenomenon. In accordance with this hypothesis, the evolution of turbulent eddies is connected with their cascade breakdown. The energy of turbulent motion is transformed to heat in the dissipation of the smallest eddies.

In the study [1], Townsend first formulated the problem of constructing a simplified turbulence model representing a system of ordered large-scale eddies, and he proposed several models based on experimental data on space—time correlations in the flow. However, the Townsend model did not sufficiently closely reflect the actual structure of the flow, and in particular it failed to consider the development of the eddies.

In the 1970s experiments involving visualization of turbulent shear flows showed [4] that such flows are much more ordered than was previously thought. It turned out that flows earlier considered to constitute "normal" turbulence have a quasiordered structure highly reminiescent of the Townsend model. Here, the eddies join together in pairs and become larger downstream. In [5], the impossibility of breaking up eddies in the case of two-dimensional turbulence was proved theoretically.

Present integral methods of calculating flows on the basis of the Prandtl concept of mixing length successfully describe the distribution of turbulent shear stresses but agree poorly with empirical data on the structure of flows. The theory of hydrodynamic stability also is unable to explain several phenomena, such as the occurrence of secondary instability in the boundary layer [6] and aeroacoustic interaction [7]. In connection with this, there is a need to construct a turbulence model which corresponds to the present level of knowledge about turbulence.

Below we present a model of the eddy structure of a flow in the case of longitudinal flow about a smooth surface, using a minimum number of assumptions in constructing the model. Here, we refine and elaborate on the conclusions reached in [8] regarding the effect of perturbations propagated across the flow on the turbulence structure.

As is known from experiments (see the surveys in [4, 7, 9]), quasiordered motion occurs in the boundary layer in the form of periodic "ejections" of slowed fluid from the viscous sublayer into the external region of the boundary layer. Zones of quasiordered motion are characterized by turbulent pulsations of high intensity and an increase in the scale of turbulence connected with the pairwise combination of eddies. Obviously, this zone does not occupy the entire cross section of the boundary layer: there is a viscous sublayer in the wall

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region, while in the external region of the boundary layer turbulence decays. As was noted in [8], it follows from similarity considerations that the distance from the wall to the internal boundary of the eddy growth zone is proportional to the thickness of the viscous sublayer, while the distance to the external boundary is proportional to the thickness of the boundary layer, i.e.,

$$k_1 \frac{\nu}{u_\tau} \leqslant y \leqslant k_2 \delta. \tag{1}$$

We will evaluate the coefficients k_1 and k_2. It was established in [10] that at y^+ = 10-12 the intensity of the pulsations of the longitudinal component of velocity reach a maximum, the asymmetry factor passes through zero, and the excess factor is minimal. This provides grounds for suggesting that it is here, on the external boundary of the viscous sublayer, that intensive growth of turbulent eddies begins. As we proposed in [8], this growth is associated with resonance interaction. We therefore take the mean value for the coefficient $k_1 = 11$. In accordance with the data in [11], the increase in the turbulence scale stops at $y/\delta = 0.22$, from which $k_2 = 0.22$.

An inequality similar to (1) also limits the zone of eddy development in the case of flow in a circular pipe, but it is necessary here to replace the thickness of the boundary layer δ by the diameter of the pipe d. In this case, the dimensionless coefficients take values $k_{1d} = 11$ and $k_{2d} = 0.11$.

If we represent a turbulent eddy in the form of a quasisolid rotating cylinder, then the frequency of rotation of such an eddy is determined by the gradient of the mean flow velocity at the point of its formation:

$$\omega = \frac{1}{2} \frac{d\langle u \rangle}{dy}. \tag{2}$$

Now we assume that the discrete eddy generates oscillations with the below frequency into the surrounding medium

$$f = \omega / 2\pi. \tag{3}$$

To approximate the universal velocity profile in the boundary layer, it is useful to use the Van Driest formula. This formula gives a single analytic expression for the entire profile [11]:

$$u^+ = \int_0^{y^+} \frac{2dy^+}{1 + \sqrt{1 + (2\varkappa y^+)^2 [1 - \exp(-y^+/A)]^2}}, \tag{4}$$

where $\varkappa = 0.4$ and $A = 26$. Then the dimensionless frequency of the oscillations generated by the eddy, formed at a distance y^+ from the wall, is determined by the relation

$$f^+ = \frac{1}{2\pi \{1 + \sqrt{1 + (2\varkappa y^+)^2 [1 - \exp(-y^+/A)]^2}\}}. \tag{5}$$

At $y^+ < 3$, the value of f^+ asymptotically approaches its maximum value

$$f^+_{\max} = 1/(4\pi). \tag{6}$$

The following assumption concerns the propagation of oscillations generated by the eddy ("long-range waves" in accordance with [12]) along a normal to the streamlines. We will assume that such waves are propagated in a manner similar to acoustic waves and are reflected from the wall which bounds the flow. The rate of propagation of the perturbations a in a first approximation is taken to be constant across the flow. Then, for certain distances between the oscillation source and the wall, stationary waves will be formed, and the amplitude of the oscillations near the source will increase sharply. If we assume that the oscillation source is located at an antinode of the stationary wave, the family of resonance frequencies can be written as follows:

$$f_n = \frac{2n - 1}{4} \frac{a}{y}, \tag{7}$$

where n = 1, 2, 3, ... It is evident from Eq. (7) that the resonance distance decreases with an increase in the frequency f_n and with a decrease in n. But since the frequency of the oscillations which occur in the flow is bounded above by Eq. (6), and since n cannot be less than unity, then, beginning with a certain distance from the wall, resonance growth of the disturbances will be impossible. This obviously occurs at the nearest boundary of the resonance zone (1), i.e., at the boundary of the viscous sublayer.

Comparing Eqs. (6) and (7) and considering (1), we obtain the rate of propagation of the oscillations:

$$a = u_\tau k_1/\pi. \tag{8}$$

Then, from (7),

$$f_n^+ = \frac{2n-1}{4\pi}\,\frac{k_1}{y^+}. \tag{9}$$

Curve 1 in Fig. 1 corresponds to Eq. (5). Its physical significance is the frequency of the oscillations generated by the eddy formed at the distance y^+ from the wall. The family of lines 2-5 is described by Eq. (9) at n = 1-4 and k_1 = 11. These are resonance frequencies corresponding to the given distance from the wall.

Let us examine the consequences of the above assumptions. The first and most important consequence is cascade growth of turbulent eddies with increasing distance from the wall.

We will follow the development of a discrete eddy during the "ejection" period. We will assume that an eddy is formed in the viscous sublayer at $y^+ < 3$. At this moment it corresponds to a point on curve 1 lying to the left of point A. Under the influence of the Zhukov buoyancy force the eddy is kept away from the wall. If we ignore the reduction in the angular velocity of the eddy due to viscous dissipation, then the process of retreat of the eddy from the wall corresponds to the horizontal line in Fig. 1. At point B this line intersects line 2.

Point B determines the distance from the wall at which the natural oscillations of the eddy are significantly amplified by resonance interaction with oscillations reflected from the wall. It is known from experiments that such interaction leads to a situation whereby two adjacent eddies begin to rotate about a common axis and merge, forming a larger eddy. The frequency of rotation of the new eddy is determined by the velocity gradient at the point of its formation. Drawing a vertical line through B, we obtain point C at the intersection with curve 1. Point C corresponds to the formed eddy.

The cascade consisting of the formation of the eddies, their movement away from the wall, and their destruction, will continue until the thickness of the shear layer is exhausted, i.e., until the energy of the oscillations reflected from the wall is sufficient to remove the eddies from the state of dynamic equilibrium.

Thus, the gradual increase in the size of an eddy is accompanied by a gradual decrease in the frequency of its rotation. It can be shown that the assumption of constancy of the rate of propagation of the oscillations a across the boundary layer leads to the conclusion that the characteristic dimensions of the eddies at the points of their formation are linearly dependent on y.

The points B, D, F, ..., at which paired union of the eddies occurs in essence determine the position of the zone of instability for eddies with a specified natural frequency f^+. Here, it is not clear whether the instability is determined by the possibility of pairwise union or whether the position of the instability zone is such that the formation of one large eddy involves the disintegration of approximately two small eddies.

The next important conclusion which follows from the proposed model of boundary turbulence consists of the fact that several energy-carrying frequencies must be distinguished in the spectrum of turbulent pulsations. In fact, it follows from the eddy development scheme shown in Fig. 1 that the natural frequencies of the oscillations generated by the eddies must be grouped near horizontal lines AB, CD, EF, ... Since the formation of a turbulent eddy is a random process, the spectral characteristic of each stage of the cascade is described by a certain stochastic distribution.

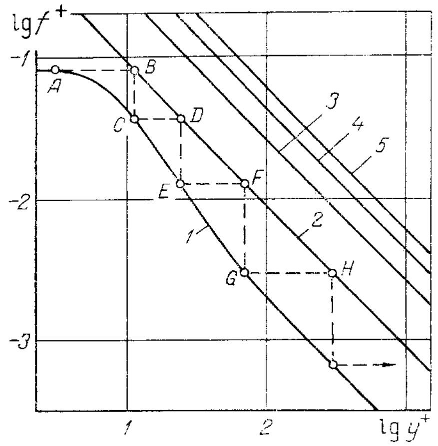

Fig. 1

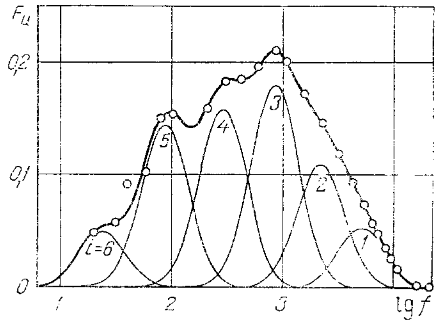

Fig. 2

Fig. 1. Theoretical model of the cascade growth of eddies in a boundary layer. Calculation: 1) from (5); 2-5) from (9) with k_1 = 11 and n = 1-4.

Fig. 2. Approximation of the spectra of the turbulent pulsations. Calculation is from (10); experimental points are from [13].

The above model only approximately describes the instability of the boundary layer in regard to perturbations propagated across the flow. To check the validity of the assumptions made, we will compare the conclusions reached with well-known experimental data.

A detailed survey was presented in [9] of studies of spectra of pulsations in a boundary layer. Many investigators have noted the presence of maxima in such spectra, but the numerical values of the dimensionless frequencies carrying the maximum energy of the oscillations, measured under different conditions, often differ from each other. This discrepancy was resolved to a significant extent in [13], where it was shown that the spectra of the pulsations have several maxima. Meanwhile, the values of the frequencies which carry the most energy remain constant across the boundary layer.

Figure 2 shows as an example experimental points obtained in [13] for Re_{**} = 3070 and y^+ = 81.5. It is evident from the figure that such a spectrum can, with a high degree of accuracy, be approximated by the expression

$$F_u = \frac{1}{\sigma\sqrt{\pi}} \sum_{i=1}^{m} c_i(y) \exp\left[-\left(\frac{\lg f - \lg f_i}{\sigma}\right)^2\right], \tag{10}$$

where i = 1, 2, ..., m are serial numbers of the maxima in the spectra. Here, from the properties of the function F_u mentioned in [13],

$$\int_{-\infty}^{\infty} F_u d(\ln f) = \int_{-\infty}^{\infty} \frac{f\langle u'^2(f)\rangle}{\langle u'^2\rangle_\Sigma} d(\ln f) = 1 \tag{11}$$

follows the normalization condition

$$\sum_{i=1}^{m} c_i(y) = 1. \tag{12}$$

The coefficients $c_i(y)$ can be regarded as the probability that the eddy located at the given point of the flow belongs to the i-th stage of the cascade.

The values of the frequencies f_i obtained in such an approximation agree well with the characteristic frequencies obtained from the proposed model by means of Fig. 1. Having made a construction which is the inverse of that made earlier, we can refine the position of the instability zone as shown in Fig. 3. It can be seen from the figure that curve 3, constructed from experimental data, deviates somewhat from curve 2, constructed from Eq. (9) with n = 1 and k_1 = 11. Thus, the rate of propagation of the perturbations a can be assumed constant only in the first approximation.

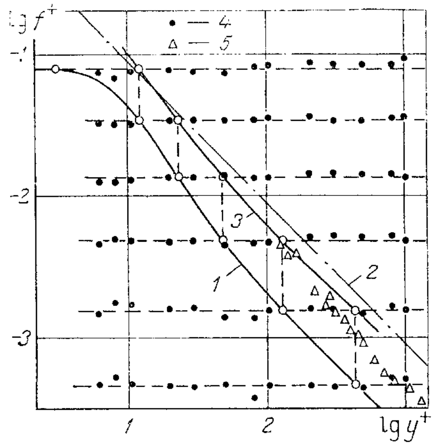

Fig. 3

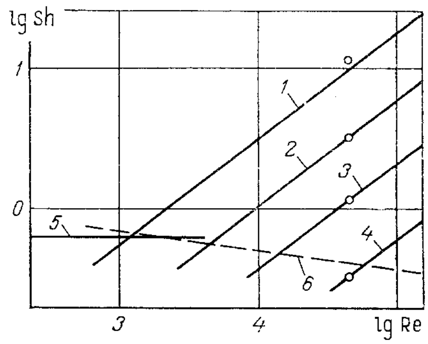

Fig. 4

Fig. 3. Refinement of the eddy growth model with allowance for the empirical data. Calculation: 1) from (5); 2) from (9) with $k_1 = 11$ and $n = 1$; 3) construction from experimental data. Experiment: 4) analysis of data from [13]; 5) [14].

Fig. 4. Dependence of the characteristic frequencies of the pulsations in a circular pipe on the Reynolds number. Calculation: 1-4; from (15) with i = 1-4; 5) (16); 6) (17). The experimental points are from [16].

It is interesting to compare the graph obtained with experimental data from [14] on the rate of propagation of "ejections" over the radius of a pipe. It can be concluded from Fig. 3 that the time of movement of the eddy from the wall to the point with the coordinate y is close to the period of the pulsations at the given point. This may signify that the oscillation carrier in "forward" motion of the eddy away from the wall is the eddy itself, and the feedback ensuring the nucleation of new eddies is provided by sound waves propagating at a velocity considerably greater than a. If such a conclusion is confirmed by later investigations, this will mean that the position of the instability zone (Fig. 3) depends on the Mach number.

There is one more important consequence of the above assumptions — an increase in the number of maxima in the pulsation spectra with an increase in the Reynolds number. Let us illustrate this conclusion using the example of flow in a circular pipe, since the frequency range of the pulsations in the boundary layer was already examined in [8]. Here we will consider the universal character of Eq. (4).

The dynamic velocity in a circular pipe is equal to [15]

$$u_\tau = v\sqrt{\lambda/8}, \qquad (13)$$

while the Strouhal number is determined by the identity

$$\mathrm{Sh} = f^+ \mathrm{Re}\,(u_\tau/v)^2. \qquad (14)$$

If we consider that the position of the instability zone (Fig. 3) is independent of Re, then the i-th maximum in the pulsation spectrum corresponds to the equation

$$\mathrm{Sh}_i = f_i^+ \mathrm{Re}\,\lambda/8. \qquad (15)$$

The values of the frequencies f^+_i in the case of a circular pipe differ somewhat from the values obtained above for the boundary layer. Using the values of f^+_i calculated from the data in [16] and the Blausius formula for λ, we can construct lines 1, 2, 3, 4, ..., in Fig. 4 corresponding to the horizontal lines in Fig. 3.

It should be noted that, in the case of laminar flow, cascade eddy growth does not occur, and the frequency of the pulsations which arise does not exceed

$$\mathrm{Sh}_{max} = \frac{\mathrm{Re}}{32\pi}\,\frac{64}{\mathrm{Re}} = \frac{2}{\pi}. \qquad (16)$$

A graph of this relation is represented by line 5 in Fig. 4.

As was noted above, with $y/d = k_{2d} = 0.11$, turbulent eddies again become stable due to the decay of pulsations caused by viscosity. Equations (1), (6), and (13) can be used to determine the lower boundary of the frequency range of the pulsations:

$$\mathrm{Sh}_{\min} = \frac{k_{1d}}{4\pi k_{2d}} \sqrt{\frac{\lambda}{8}}. \quad (17)$$

The graph of Eq. (17) is represented by line 6 in Fig. 4. Below this boundary the intensity of the pulsations quickly decreases, which is manifest in a decrease in the coefficients $c_m(y)$. It can be seen from the graph that the number of maxima in the spectrum between Sh_1 and Sh_{min} increases discretely with an increase in the Reynolds number.

The maximum frequency of the pulsations f^+_1 in [16] was clearly recorded only in the transverse component of the velocity fluctuations. Also, as can be seen from Fig. 3, up to the second union ($y^+ < 30$) the turbulent eddies do not significantly affect the character of the velocity diagram. Both of these phenomena are evidently connected with the fact that in this region the eddies carrying the maximum frequency f^+_1 remain nearly two-dimensional. Thus, it can be concluded that the presence of a minimum of two stages of a cascade is necessary for the formation of a stable turbulence structure.

This conclusion is also supported by the pattern of the transition from laminar to turbulent flow. It is evident from Fig. 4 that a formally single-stage turbulent cascade may arise beginning with Re ≈ 1200. In fact, flow remains laminar in the pipe up to Re ≈ 2300 and the transition to turbulent flow is completed at Re ≈ 4000, which corresponds to the occurrence of a two-stage cascade (Fig. 4).

NOTATION

y, distance from the wall bounding the flow; δ, thickness of the turbulent boundary layer; d, pipe diameter; $\langle u \rangle$, u', mean and fluctuation components of longitudinal velocity at a point of the flow; $\langle u'^2 \rangle_\Sigma$, $\langle u'^2(f) \rangle$, averaged square of fluctuation velocity and its spectral density; u_τ, dynamic velocity; v, mean velocity across the pipe; ω, angular velocity of eddy; f, frequency of oscillations; ν, kinematic viscosity of the fluid; λ, Darcy coefficient; k_1, k_2, k_{1d}, k_{2d}, $c_i(y)$, dimensionless coefficients; σ, dispersion characteristic of the velocity pulsation spectra; $y^+ = yu_\tau/\nu$, dimensionless distance from wall; $u^+ = \langle u \rangle/u_\tau$, dimensionless velocity; $f^+ = f\nu/u^2_\tau$, dimensionless frequency of the pulsations; Re_{**}, Reynolds number determined from the momentum thickness in the boundary layer; $Re = vd/\nu$, Reynolds number for pipe flow; $Sh = fd/v$, Strouhal number; $F_u = f\langle u'^2(f) \rangle / \langle u'^2 \rangle_\Sigma$, spectral function.

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ATTENUATION OF LOCAL GAS SWIRLING IN A CHANNEL OF ANNULAR CROSS SECTION

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The attenuation of local gas swirling in smooth channels of annular cross section with $r_{in}/r_o = 0.89$-0.95 is investigated. Calculating equations satisfactorily describing the experimental data are obtained.

We consider the established flow of a viscous incompressible liquid in a relatively thin cylindrical channel of annular cross section with a ratio of inside to outside radii $r_{in}/r_o \approx 0.9$ behind a swirling device at moderate velocities.

Under the assumption of constancy of all the stream parameters along the circumference of the channel, isotropy of the turbulent properties, the absence of secondary flows, and $\partial^2V/\partial x^2 \ll \partial^2V/\partial r^2$, the equations of conservation of momentum [1], written through the shear stresses, have the form

$$\frac{\partial P}{\partial x} = \frac{1}{r}\frac{\partial (r\tau_{rx})}{\partial r}, \tag{1}$$

$$\rho V_x \frac{\partial V_\theta}{\partial x} = \frac{1}{r^2}\frac{\partial (r^2\tau_{r\theta})}{\partial r}. \tag{2}$$

In connection with the fact that the variation along the channel of the average swirling over the cross section V_θ/V_x is of primary interest for engineering applications, it is expedient to convert to parameters averaged over a cross section in Eqs. (1) and (2).

Taking the relative thickness of the channel as small for the case under consideration, $\delta/r_o \leq 0.1$, where $\delta = r_o - r_{in}$, taking the shear stresses at the inner and outer walls as equal in absolute value and opposite in sign, $\tau_{wo} = -\tau_{win} = -\tau_w$, as well as $\partial/\partial r(\partial p/\partial x) = 0$, we integrate Eqs. (1) and (2) over r from r_{in} to r_o, having preliminarily multiplied the first by r and the second by r^2. After integration we average the terms of the second equation over the channel cross section. As a result, we obtain

$$dP/dx = -2\tau_{wrx}(r_o + r_{in})/(r_o^2 - r_{in}^2) = -2\tau_{wrx}/\delta = -2(\tau_w/\delta)\cos\varphi, \tag{3}$$

$$\rho\int_{r_{in}}^{r_o} r^2V_x(dV_\theta/dx)\,dr \Big/ \int_{r_{in}}^{r_o} r\,dr = -2\tau_{wr\theta}(r_o^2 + r_{in}^2)/(r_o^2 - r_{in}^2)$$

or, replacing $(r_o^2 + r_{in}^2)$ by $(r_o + r_{in})^2/2$ with an error of no more than 1%,

$$\rho V_x(dV_\theta/dx) = -2(\tau_w/\delta)\sin\varphi. \tag{4}$$

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